MATHEMATICS TEST

Duration: 4 hours

EXERCISE I: (4 marks)

f is the application of $\mathbb{C}$ in $\mathbb{C}$ defined by:
$$f(z) = z^3 - 2(2 + 3i)z^2 + (-6 + 11i)z + 7 + i$$

1. Show that the equation $f(z) = 0$ admits a purely imaginary root z_1 that we will determine. (0.75mark)

2. 2.1. Determine three complex numbers a, b et c such as: for all z belonging to $\mathbb{C}$,
   $$f(z) = (z - i)(az^2 + bz + c)$$ (1mark)

   2.2. Solve the equation $f(z) = 0$. Call the solutions z_2 and z_3 such as $|z_3| > |z_2|$
   (0.75mark)

3. The complex plane is the orthonormal coordinate system (O ; $\vec{u}, \vec{v}$). A, B and C are points with respective affix plane $z_A = i$, $z_B = 1 + i$ and $z_C = .3 + 4i$.

   S is the direct plane similitude which leaves point A invariant and transforms C into B.

3.1. Determine the equation associated with S. (1mark)

3.2. Determine the geometric elements of S. (0.5 mark)

EXERCISE II: (6 marks)

A / Let P be the polynomial defined by $P(x) = x^3 + ax + b$ with a and b real.
1- Study the variations of P according to the values of a and draw up your variation table. (1mark)

2- We assume a strictly negative.

   a) Prove that the polynomial p admits a unique real root if $4a^3 + 27b^2 > 0$ (0.75pt)

   b) Prove that the polynomial p admits three real roots if and only if $4a^3 + 27b^2 < 0$

   (0.75mark)

   c) Prove that the polynomial p admits two distinct real roots α and β that we will determine such that $P(x) = (x - \alpha)^2(x - \beta)$ if $4a^3 + 27b^2 = 0$
   (0.75mark)

B / In this part $P(x) = x^3 - 2x + 4$

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Rue 1504, BP 35364, Yaoundé Omnisport Mfandena
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1- Show that P has a real root. (0.5mark)

2- a) Prove that if U and V are two complex numbers such that $U^3 + V^3 = -4 \ and \ UV = \frac{2}{3}$
then $U + V$ is a root of P. (1mar)

b) Deduce that $U^3 \ and \ V^3$ are solutions of the equation $x^2 + 4x + \frac{8}{27} = 0$ (1mark)

EXERCISE III: (5 marks)

The Youta family owns a printing house. The number of books printed in previous years is given by the following table:

year	2007	2008	2009	2010	2011	2012
Position of the year x_i	1	2	3	4	5	6
Number of books printed in thousand y_i	15,35	15,81	16,44	16,75	17,19	17,30

1- Write an equation of the regression line of y on x. It is assumed that this trend remains observed. 2marks

2- From 2012, we see that production drops by 4% each year and the printing press will be declared bankrupt when production drops below 6,000 pounds.

In what year will the bankruptcy of this printing press be declared? 3marks

EXERCISE IV: (5 marks)

An urn contains two black balls and one white ball. A ball is drawn at random from the urn. If the drawn ball is white the game is over. If it is black, put it back in the ballot box and draw a ball again. If the new ball drawn is white, the game is over, otherwise it is returned to the ballot box and a new ball is drawn, and so on. But at most n draws are made. That is to say at the nth draw the game is over regardless of the ball drawn.

Let X be the random variable which determines the number of draws made by a player for the game to end.

Under the assumption of equiprobability

1- a) Define the probability law of the random variable X (we will express Pi = P (X = i) as a function of i). (1.5mark)

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b) Check that $\sum_{i=1}^{n} P_i = 1$ (1.5mark)

2- Show that the mathematical expectation is $E(X) = \frac{1}{3}f'_n\left(\frac{2}{3}\right) + n(\frac{2}{3})^n$ knowing that we have the function $f_n(x) = -1 + \sum_{k=1}^{n} x^k$ with n a strictly positive integer. (2marks)

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